

PEIERLS INSTABILITY OF VORTEX TUBES.

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Abstract

We discuss the possibility of a new low temperature instability of vortex tubes to a 'folded' state, driven by the coupling to the normal electron states inside the cores. The basic mechanism is that a bended tube creates an effective potential for the electrons, which destabilizes the tube in a way reminiscent of the usual Peierls instability.

Résumé: Nous discutons la possibilité d'une nouvelle instabilité 'ondulatoire' des tubes de vortex, induite par le couplage aux états électroniques normaux au coeur de ceux-ci. Le mécanisme responsable de cette instabilité est le fait qu'un tube ondulé crée sur les électrons qui y sont confinés un potentiel effectif attractif. L'instabilité qui en résulte s'apparente à l'instabilité de Peierls.

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SUR UNE INSTABILITE DE PEIERLS DE VORTEX

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La physique des lignes de vortex dans les supraconducteurs a attiré une attention considérable ces dernières années. Les propriétés *collectives* de ces objets, en particulier en présence de désordre, ont été explorées en grand détail (Blatter et al., Fisher et al.). Simultanément, de nouvelles techniques expérimentales ont été développées pour sonder la structure microscopique de ces vortex (Maggio-Aprile et al.).

Le but de cette note est d'explorer les conséquences d'un effet, discuté récemment par Goldstone et Jaffe, qui est le suivant: un électron confiné dans un tube courbé ressent un potentiel effectif négatif proportionnel au carré de la courbure locale (Eq.(1)). Ceci pourrait avoir comme conséquence intéressante de rendre les vortex dans les supraconducteurs à faible longueur de cohérence instables vis à vis de petites modulations périodiques. Le mécanisme de cette instabilité est le couplage entre géométrie du vortex et degrés de liberté longitudinaux des électrons normaux confinés dans le coeur du vortex (Caroli et al.). Une distortion périodique du vortex crée donc un potentiel de période double, qui abaisse considérablement l'énergie du gaz électronique unidimensionnel (Eq.(3)). En revanche, cette distortion coûte une énergie 'élastique' due à l'allongement du vortex et à la modification du champ magnétique environnant (Eq.(4)). De façon tout à fait similaire à l'instabilité de Peierls, cette compétition conduit à l'apparition d'une modulation de vecteur d'onde k_f , et d'amplitude δ dont l'ordre de grandeur vaut, par rapport à la longueur de cohérence ξ , $\frac{\delta}{\xi} \simeq (k_f \xi)^{-2}$. Cette instabilité ne conduit à des effets appréciables que si $k_f \xi \lesssim 1$, ce qui assure que le gaz électronique est bien unidimensionnel et que le couplage avec la conformation externe du vortex est important. De plus, la température doit être très faible, pour éviter que les fluctuations thermiques (divergentes avec la longueur du vortex – voir Eq. (6)) ne masque la modulation périodique discutée ici. Pour des vortex en interaction, la longueur effective des vortex est fixée par le module de cisaillement du réseau d'Abrikosov, et est de l'ordre du micron. Pour $T = 10^{-2}$ K, les fluctuations thermiques sont de l'ordre de quelques Angstroms, déjà comparables à $(k_f \xi)^{-2}$. Une instabilité analogue pourrait aussi apparaître pour d'autres objets unidimensionnels, comme les dislocations.

The physics of vortex lines in superconductors is fascinating. Many new aspects have been discussed in the recent years in connection with the high temperature materials, and various phases discovered (vortex liquid, vortex glass, etc. (Blatter et al., Fisher et al.)), unveiling interesting collective properties of these lines defects. Simultaneously, new experimental techniques (tunneling microscopy) have been developed to investigate the microscopic structure of the vortex cores (Maggio-Aprile et al.) , and to probe the normal electron states within these cores (Caroli et al.).

The aim of this note is to show that individual vortex lines should, under certain conditions, undergo a Peierls like transition to a ‘folded’ state, driven by the coupling between these normal electron states within the core and the conformation of the core. The idea is very simple and relies on the fact that the effective Schrodinger equation describing an electron confined in a distorted tube contains a curvature induced negative potential term. As shown by Goldstone and Jaffe, in the limit of small, slowly varying curvature, the Schrodinger equation reads:

$$-\frac{\hbar^2}{2m}[\nabla_{\perp}^2\phi] - \frac{\hbar^2}{2m}\frac{\partial^2\phi}{\partial s^2} - \frac{\hbar^2}{8m}[\kappa(s)]^2\phi = E\phi \quad (1)$$

where s is the curvilinear coordinate along the distorted tube, $\kappa(s)$ the local curvature of the tube, and ∇_{\perp} is the Laplacian in the directions transverse to the tube. Writing $\phi(s, \vec{x}_{\perp}) = \psi(\vec{x}_{\perp})\varphi(s)$, one finds a one dimensional Schrodinger equation for $\varphi(s)$, with an effective attractive potential $\mathcal{V}(s) = -\frac{\hbar^2}{8m}[\kappa(s)]^2$. As emphasized by Goldstone and Jaffe, bends are thus sufficient to create bound states in a tube.

Now consider the one dimensional gas of normal electrons living in a vortex tube. If the tube is periodically modulated in a given direction, say $\vec{x}_{\perp}(s) = (\delta \cos(Qs), 0)$, the effective potential is given by:

$$\mathcal{V}(s) = -\frac{\hbar^2}{16m}\delta^2Q^4[1 + \cos(2Qs)] \quad (2)$$

Note that the curvature δQ^2 cannot exceed ξ^{-1} , where ξ the size of the vortex core. Furthermore, Goldstone and Jaffe’s description only hold if the transverse boundary conditions confining the electron are sufficiently ‘sharp’. Since the ‘width’ of the vortex walls is also of order ξ , this imposes that $\delta \gtrsim \xi$. This condition will restrict our results to the case of *short coherence length* superconductors, such that $k_f\xi$ is not too large, where k_f is the Fermi momentum.

The change of energy (per unit length) of the electron gas to this periodic potential is given by the classical expression (Kagoshima et al.):

$$\Delta E_1 = -\rho\frac{\hbar^2}{16m}\delta^2Q^4 - \rho\chi(2Q)[\frac{\hbar^2}{16m}\delta^2Q^4]^2 \quad (3)$$

where ρ is the linear density of electrons within the core. In the case where $k_f \xi \lesssim 1$, $\chi(Q)$ the one dimensional polarisation function, which diverges at $Q = 2k_f$. Eq. (3) means that the effective free energy of the line will contain *negative* terms proportional to the curvature and the square of the curvature.

On the other hand, the vortices have a *positive* line tension c_{44} , reflecting the fact that the superconducting order parameter is modified around the vortex. For a single vortex line, one finds $c_{44} \simeq (\frac{\phi_0}{4\pi\lambda})^2$, with ϕ_0 the flux quantum, λ the penetration depth. The energy per unit length of the distorsion then reads:

$$\Delta E_2 = +\frac{1}{4}c_{44}\delta^2 Q^2 \quad (4)$$

c_{44} actually depends on the wavevector of the distorsion (Brandt et al.), and on the fact that other vortices are present. In particular, one also expects a positive curvature term of the order of $c_{44}\delta^2 Q^2 (Q\xi)^2$ to appear in Eq. (4).

Let us first look at long wavelength modulations $Q \ll k_f, \xi^{-1}$, such that $\chi(2Q) \simeq 0$. Comparison between Eqs. (3) and (4) reveals that modes with $Q > Q^*$ are unstable, with

$$\rho \frac{\hbar^2}{2m} Q^{*2} = 2c_{44}(Q^*) \quad (5)$$

Taking $\xi = 1nm$, an electron density $\rho = 1 nm^{-1}$, a Fermi energy $\frac{\hbar^2}{2m}k_f^2 = 100K$, and $c_{44} \simeq 50Knm^{-1}$ (corresponding to $\lambda = 1\mu m$), one finds that Q^* is $\simeq k_f$, which means that the instability will actually be of short wavelength, and $\chi(2Q)$ cannot be neglected. The instability thus takes place where $\chi(2Q)$ is maximum, i.e. right at $Q = k_f$ (and not $2k_f$ as in the usual Peierls instability). The resulting value of the modulation δ requires higher order terms in Eqs.(3) and (4), but presumably is of order $(k_f^2 \xi)^{-1}$. Correspondingly, a gap in the longitudinal electron states for $k = k_f$ will appear. Note that $\frac{\delta}{\xi} \simeq (k_f \xi)^{-2}$, which in turn imposes the condition $k_f \xi \lesssim 1$ mentioned above. For larger ξ , the electron gas inside the core loses its one-dimensional nature, and the influence of the modulated boundary condition becomes quickly irrelevant.

The above discussion was restricted to zero temperature. When the temperature grows, $\chi(2k_f)$ becomes finite and this reduces the tendency to instability. Nevertheless, the first term in ΔE_1 still makes the high Q modes unstable, due to the basic effect that the curvature pushes down all the electronic states. However, the effect discussed here should rapidly become unobservable because of the thermal fluctuations of the vortex. For an isolated vortex, the thermal displacement is given by (Blatter et al., Fisher et al.):

$$\delta_T \propto \sqrt{\frac{\mathcal{L}T}{c_{44}}} \quad (6)$$

where \mathcal{L} is the length of the vortex. For interacting vortices, \mathcal{L} is replaced by a length determined by the shear modulus of the vortex lattice, and is of the order of $1\mu m$. For $T = 10^{-2}$ K, one thus finds δ_T of the order of a few Angstroms, already comparable to $(k_F^2\xi)^{-1}$. Hence, the effect suggested here might only be observable at very low temperatures. A similar instability might also be relevant for other linear defects, such as dislocations.

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REFERENCES:

For an extensive review, see G. Blatter, M. V. Feigel'man, V. B. Geshkenbin, A. I. Larkin and V.M. Vinokur, 1994, 'Vortices in high temperature superconductors', Rev. Mod. Phys. **66** 4.

See e.g. E.H. Brandt, U. Essmann, 1987, 'The flux-line lattice in Type-II superconductors', Phys. Status Solidi **B 144**, 13

C. Caroli, P. G. de Gennes, J. Matricon, 1964, 'Bound fermion states on a vortex line in Type-II superconductors', Phys. Letters **9**, 307

D.S. Fisher, M.P.A. Fisher, D.A. Huse, 1991, 'Thermal fluctuations, quenched disorder, phase transitions and transport in Type-II superconductors' Phys. Rev. **B 43**, 130

J. Goldstone, A. Jaffe, 1992, 'Bound states in twisted tubes', Phys. Rev. **B 45**, 14100

See e.g. 'One Dimensional Conductors', by S. Kagoshima, H. Nagasawa, T. Sambongi, Solid-State Science, Springer-Verlag (1991).

I. Maggio-Aprile, Ch. Renner, A. Erb, E. Walter, O. Fischer, 1995, 'Direct vortex lattice imaging and tunneling spectroscopy of flux lines in YBACuO', Phys. Rev. Letters, **75**, 2754